New Formulation for Distribution System State Estimation

Paulo M. De Oliveira-De Jesus
Energy Institute
Simon Bolivar University
Caracas, Venezuela
pdeoliveira@usb.ve

Andrés Rojas Quintana
Conversion and Energy Delivery Department
Simon Bolivar University
Caracas, Venezuela
arojasq@usb.ve

***Abstract*—This paper presents a new formulation for system state estimation of passive electric distribution networks. The fundamental idea discussed here is how to obtain the state of the system at maximum demand condition using three sources of information: 1) a quasi-symmetric matrix called TRX representing network structure and topology, 2) power measurements at main feeder disconnection devices and reclosers installed in the network, and 3) Energy measurements and estimated load curve of aggregate users at distribution transformers. The method is formulated with real variables for single-phase positive sequence radially operated networks. Results of the application of proposed methodology are presented using the well known IEEE 4-Node test system.**

I. INTRODUCTION

Seminal contributions on System State Estimation (SSE) procedures applied to meshed electrical networks were introduced by Schweppe in the early 70s [1], [2], [3]. The basic formulation is based on a minimum least squares nonlinear optimization problem, solved using Newton approach. The SEE problem applied to Electric Distribution Systems was extended by [4], using three phase model solved also with the iterative Newton method. Baran [5] and [6], proposed solve the problem using the current branch measurement instead of power injection formulae, transforming power flow measurements in their equivalent currents values.

In our knowledge, System State Estimation in electric distribution systems methodologies have been applied in the past using three-phase formulations from standard power flow equations. This paper presents a new formulation for the solution of state of the system estimation based upon the direct relation between injected currents and voltage drops in radial exploited networks given by a unique matrix called TRX.

This paper is organized as follows: Section II addresses the methodology for single-phase balanced distribution systems, Section III described the mathematical formulation of the problem based on minimum least squares equations. Case study is shown in Section IV and conclusions are drawn in Section V. Demonstration of TRX matrix structure is provided in Appendix A.

II. PROPOSED METHODOLOGY

A. Data Preparation

The State Estimator requires the preparation of the following data: Network model via a unique quasi-symmetric matrix, active energy and power factor measured in all load demands and voltage and current measurements at main feeder in substation.

1) The TRX Matrix: In order to build the TRX matrix it is required to know branch per unit impedances stated as $\mathbf{Z}$ corresponding to a vector of series line impedance or transformer, positive sequence resistance and reactance for each branch of the network.

$$\mathbf{Z} = \left[\begin{array}{ccccc} \overline{Z}_{01} & ... & \overline{Z}_{ij} & ... & \overline{Z}_{mn} \end{array} \right] \tag{1}$$

where,

$$\overline{Z}_{ij} = R_{ij} + jX_{ij} \quad i,j = 1,...,n \quad i \neq j \tag{2}$$

For simplicity, shunt admittances are not considered in the balanced approach. Figure 1 shows a radial distribution network with $n+1$ buses, and n branches and a single voltage source at the root bus 0. Branches are organized according to an appropriate numbering scheme (list), whose details can be found in [7].

Relationship between injected currents $\mathbf{I}$ and branch currents $\mathbf{J}$ is set through an upper triangular matrix $\mathbf{T}$ accomplishing the Kirchhoff Current Laws (KCL) [8], [9] as follows:

$$\mathbf{J} = -\mathbf{T} \cdot \mathbf{I} \tag{3}$$

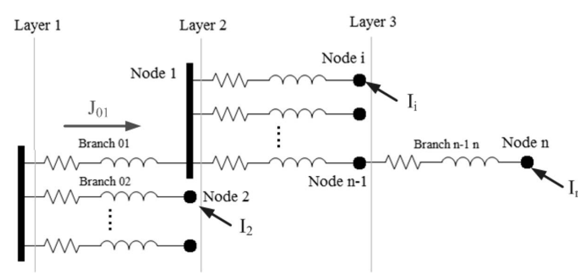

Fig. 1. Branch and bus numbering of a radial distribution network

Then, the $\mathbf{TRX}$ matrix is given by::

$$\mathbf{TRX} = \left[\begin{array}{cc} \mathbf{T}^T\mathbf{RT} & -\mathbf{T}^T\mathbf{XT} \\ \mathbf{T}^T\mathbf{XT} & \mathbf{T}^T\mathbf{RT} \end{array} \right] = \left[\begin{array}{cc} \mathbf{A} & -\mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{array} \right] \tag{4}$$

where triangular matrix $\mathbf{T}$ is given by KCL laws and impedance matrices $\mathbf{R}$ and $\mathbf{X}$ are given by:

$$\mathbf{R} = \Re e\{diag(\mathbf{Z})\}; \quad \mathbf{X} = Im\{diag(\mathbf{Z})\} \tag{5}$$

978-0-7695-4882-1/12 $26.00 © 2012 IEEE
DOI 10.1109/Andescon.2012.11

Demonstration of how TRX matrix is structured is presented in Appendix A.

2) Measurements - Bus Energy Data: Active energy in pu is measured in each bus during a month (T_m=720 hours)

$$\mathbf{W_M} = \left[\begin{array}{ccccc} W_{M1} & ... & W_{Mi} & ... & W_{Mn} \end{array} \right] \quad (6)$$

3) Measurements - Main Feeder and Reclosers: Voltage and delivered active and reactive powers measured by main feeder protection devices and reclosers installed in different branches of the grid are collected in the following measurement vectors:

$$\mathbf{V_M} = \left[\begin{array}{ccccc} V_{M0} & ... & V_{Mi} & ... & V_{Mn} \end{array} \right] \quad (7)$$

$$\mathbf{P_M} = \left[\begin{array}{ccccc} P_{M01} & ... & P_{Mij} & ... & P_{M(n-1)n} \end{array} \right] \quad (8)$$

$$\mathbf{Q_M} = \left[\begin{array}{ccccc} Q_{M01} & ... & Q_{Mij} & ... & Q_{M(n-1)n} \end{array} \right] \quad (9)$$

4) Additional Data: Power factor angle at each load bus φ_{Di} is given by:

$$\varphi_{\mathbf{D}} = \left[\begin{array}{ccccc} \varphi_{D1} & ... & \varphi_{Di} & ... & \varphi_{Dn} \end{array} \right] \quad (10)$$

Load factor at each load bus L_{Fi} is given by:

$$\mathbf{L_F} = \left[\begin{array}{ccccc} L_{F1} & ... & L_{Fi} & ... & L_{Fn} \end{array} \right] \quad (11)$$

Load factors are the relationships between average and maximum load demand at each bus under given period.

III. System State Estimation Problem Formulation

In this section does present the formulation of the optimization problem. The objective is to minimize least square error in all measurements subject to the direct relationship between the state of the system and the injected currents:

$$\min \frac{1}{2} \left(\mathbf{z} - h(\mathbf{x})\right)^T \mathbf{W} \left(\mathbf{z} - h(\mathbf{x})\right) \quad (12)$$

subject to:

$$\left[\begin{array}{c} \mathbf{V_x} \\ \mathbf{V_y} \end{array} \right] = \left[\begin{array}{c} \mathbf{V_{x0}} \\ \mathbf{V_{y0}} \end{array} \right] + \left[\begin{array}{cc} \mathbf{A} & -\mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{array} \right] \left[\begin{array}{c} \mathbf{I_x} \\ \mathbf{I_y} \end{array} \right] \quad (13)$$

Demonstration of how to obtain (13) is presented in Appendix A.

A. Measurement Vector

We have three types of measurements: energy in load buses and voltages and power measurements coming from SCADA registers in protection devices either on substation (main feeder) or reclosers located at trunk feeders. The $\mathbf{z}$ is the measurement vector defined by:

$$\mathbf{z} = \left[\begin{array}{cccc} \mathbf{V}_M & \mathbf{P}_M & \mathbf{Q}_M & \mathbf{W}_M \end{array} \right] \quad (14)$$

B. The Estimated State of the System Vector

The unknown values of the optimization problem, ie. the estimate value of voltages of all nodes of the network (real an imaginary parts) $\mathbf{x}$ are expressed as by:

$$\mathbf{x} = \left[\begin{array}{cccc} V_{x0} & V_{y0} & \mathbf{V_x} & \mathbf{V_y} \end{array} \right]^T \quad (15)$$

and voltage at reference bus $V_0 = V_{x0} + jV_{y0}$, and $\mathbf{V}$=$\mathbf{V_x} + j\mathbf{V_y}$,

C. The Estimated Measurements

The measurement error vector is denoted as $\mathbf{z} - h(\mathbf{x})$. Injected currents are written as function of calculated nodal energies.

$$I_{xi} = \frac{P_i V_{xi} + Q_i V_{yi}}{(V_{xi})^2 + (V_{yi})^2} \quad i = 1, ..., n \quad (16)$$

$$I_{yi} = \frac{-Q_i V_{xi} + P_i V_{yi}}{(V_{xi})^2 + (V_{yi})^2} \quad i = 1, ..., n \quad (17)$$

Where injected active P_i and reactive Q_i powers are expressed as function of calculated active energies W_{Ci}, load factor L_{Fi} and power factor angle φ_{Di}

$$P_i = -\frac{W_{Ci}}{720 \cdot L_{Fi}} \quad i = 1, ..., n \quad (18)$$

$$Q_i = P_i \cdot \tan \varphi_{Di} \quad i = 1, ..., n \quad (19)$$

Then, $h(\mathbf{x})$ is the estimation of given measurements as function of the state of the system vector defined by:

$$h(\mathbf{x}) = \left[\begin{array}{ccc} \mathbf{V}_C & \mathbf{J_C} & \mathbf{W_C} \end{array} \right] \quad (20)$$

where elements of $\mathbf{V}_C$ are given by:

$$V_{Ck} = \sqrt{V_{xk}^2 + V_{yk}^2} \quad (21)$$

k is a busbar with voltage measurement.

Branch currents are obtained using:

$$(\mathbf{J_x} + j\mathbf{J_y}) = -\mathbf{T} \cdot (\mathbf{I_x} + j\mathbf{I_y}) \quad (22)$$

$$\overline{J}_{ij} = J_{xij} + jJ_{yij} \quad (23)$$

Then, active powers at given branch ij: is:

$$P_{Cij} = V_{xi} \cdot J_{xij} + V_{yi} \cdot J_{yij} \quad (24)$$

and, branch reactive powers:

$$Q_{Cij} = V_{yi} \cdot J_{xij} - V_{xi} \cdot J_{yij} \quad (25)$$

where elements of $\mathbf{W}_C$ are given by:

$$W_{Ck} = -T_m \cdot L_{Fk} \cdot (V_{xk} \cdot I_{xk} + V_{yk} \cdot I_{yk}) \quad (26)$$

k is a busbar with of aggregate active energy measurements

IV. Case Study

In order to illustrate and evaluate the proposed method, the *IEEE 4 bus test system* was used. The proposed SEE formulation was implemented using Solver application under Microsoft Excel Program and it could be requested to authors.

IEEE 4 Bus Distribution model is detailed in [10]. As seen in Figure 2, only one load is connected to the system. Power measurements are given at main feeder. All simulations considered four-wire configuration and a step-down transformer 6MVA 12.47/4.16kV under wye-wye connection.

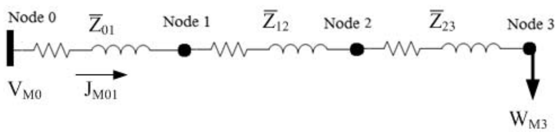

Fig. 2. IEEE 4 Node radial distribution network

The SSE problem is solved assuming a system voltage bases $V_{Base}^{High} = 12.47$kV for the medium voltage network, $V_{base}^{Low} = 4.16$kV for the low-voltage network. Power system base is $S_{base} = 6MVA$. Impedance bases are $Z_{base}^{High} = 25.92$ ohms and Z_{base}^{Low} =2.8843 ohms.

1) TRX Matrix: The positive impedance sequence matrix of the IEEE 4-node test system $\mathbf{Z}$ is given by:

$$\mathbf{Z} = \begin{bmatrix} \overline{Z}_{01} \\ \overline{Z}_{12} \\ \overline{Z}_{23} \end{bmatrix} = \begin{bmatrix} 0.0045 + j0.0092 \\ 0.0100 + j0.0600 \\ 0.0502 + j0.1029 \end{bmatrix}$$

The upper triangular matrix $\mathbf{T}$ is given by:

$$\mathbf{T} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

and the $\mathbf{A}$ and $\mathbf{B}$ matrices are:

$$\mathbf{A} = \begin{bmatrix} 0.0045 & 0.0045 & 0.0045 \\ 0.0045 & 0.0145 & 0.0145 \\ 0.0045 & 0.0145 & 0.0647 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0.0092 & 0.0092 & 0.0092 \\ 0.0092 & 0.0692 & 0.0692 \\ 0.0092 & 0.0692 & 0.1721 \end{bmatrix}$$

Then the $\mathbf{TRX}$ matrix is defined as

$$\mathbf{TRX} = \left[\begin{array}{ccc|ccc} 0.45 & 0.45 & 0.45 & -0.92 & -0.92 & -0.92 \\ 0.45 & 1.45 & 1.45 & -0.92 & -6.92 & -6.92 \\ 0.45 & 1.45 & 6.47 & -0.92 & -6.92 & -17.21 \\ \hline 0.92 & 0.92 & 0.92 & 0.45 & 0.45 & 0.45 \\ 0.92 & 6.92 & 6.92 & 0.45 & 1.45 & 1.45 \\ 0.92 & 6.92 & 17.21 & 0.45 & 1.45 & 6.47 \end{array}\right] \times 10^{-2}$$

2) Measurements: The IEEE 4 bus system has a maximum load demand of 5.4MW ($cos\varphi_{D3}$=0.9 lag). Assuming L_{F3}=0.9, active energy in p.u. registered during a month is:

$$\mathbf{W_M} = \begin{bmatrix} W_{M3} \end{bmatrix} = \begin{bmatrix} 582 \end{bmatrix}$$

Voltages at substation is:

$$\mathbf{V_M} = \begin{bmatrix} V_{M0} \end{bmatrix} = \begin{bmatrix} 1.00 \end{bmatrix}$$

Active and reactive powers at main circuit breaker are

$$\mathbf{P_M} = \begin{bmatrix} P_{M01} \end{bmatrix} = \begin{bmatrix} 0.98 \end{bmatrix}$$

$$\mathbf{Q_M} = \begin{bmatrix} Q_{M01} \end{bmatrix} = \begin{bmatrix} 0.69 \end{bmatrix}$$

Then, the measurement vector $\mathbf{z}$ is:

$$\mathbf{z} = \begin{bmatrix} V_{M0} & P_{M01} & Q_{M01} & W_{M3} \end{bmatrix}$$

$$\mathbf{z} = \begin{bmatrix} 1.00 & 0.98 & 0.69 & 582 \end{bmatrix}$$

assuming the following $\mathbf{W}$ matrix:

$$\mathbf{W} = \begin{bmatrix} 100000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

Injected powers are obtained form aggregate active energy measurements from users using approximate load curve patterns and power factor values. Hence, the best accuracies are assigned to measurements at substation and reclosers and lower accuracies are assigned to energy measurements.

3) The Optimization Problem: Given the measurement vector $\mathbf{z}$, we define the $\mathbf{h}(\mathbf{x})$ vector as:

$$\mathbf{h}(\mathbf{x}) = \begin{bmatrix} V_{C0} & P_{C01} & Q_{C01} & W_{C3} \end{bmatrix}$$

where the state of the system are all voltages in cartesian form:

$$\mathbf{x} = \begin{bmatrix} V_{x0} & V_{y0} & V_{x1} & V_{y1} & V_{x2} & V_{y2} & V_{x3} & V_{y3} \end{bmatrix}$$

The optimization problem is set by minimizing global objective (weighted least squares error):

$$\min\left(\mathbf{z} - h(\mathbf{x})\right)^T \mathbf{W} \left(\mathbf{z} - h(\mathbf{x})\right)$$

Subject To:

$$\begin{bmatrix} V_{x1} \\ V_{x2} \\ V_{x3} \\ V_{y1} \\ V_{y2} \\ V_{y3} \end{bmatrix} = \begin{bmatrix} V_{x0} \\ V_{x0} \\ V_{x0} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{A} & -\mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{bmatrix} \begin{bmatrix} I_{x1} \\ I_{x2} \\ I_{x3} \\ I_{y1} \\ I_{y2} \\ I_{y3} \end{bmatrix}$$

$$I_{x1} = I_{x2} = I_{y1} = I_{y2} = 0$$

$$I_{x3} = -\frac{W_{C3}}{T_m \cdot L_{F3}}\left(\frac{V_{x3} + \tan\varphi_{D3} \cdot V_{y3}}{(V_{x3})^2 + (V_{y3})^2}\right)$$
$$I_{y3} = -\frac{W_{C3}}{T_m \cdot L_{F3}}\left(\frac{-\tan\varphi_{D3} \cdot V_{x3} + V_{y3}}{(V_{x3})^2 + (V_{y3})^2}\right)$$

$$V_{C0} = V_{x0}$$

$$P_{C01} = -V_{x0} \cdot I_{x3} - V_{y0} \cdot I_{y3}$$

$$Q_{C01} = -V_{y0} \cdot I_{x3} + V_{x0} \cdot I_{y3}$$

$$W_{C3} = -T_m \cdot L_{F3} \cdot (V_{x3} \cdot I_{x3} + V_{y3} \cdot I_{y3})$$

4) The SEE Problem Solution: Solving the optimization model set up above using Excel's Solver, solution is given by:

$$\mathbf{x} = \begin{bmatrix} V_{x0} \\ V_{x1} \\ V_{x2} \\ V_{x3} \\ V_{y0} \\ V_{y1} \\ V_{y2} \\ V_{y3} \end{bmatrix}^T = \begin{bmatrix} 1.0000 \\ 0.9893 \\ 0.9381 \\ 0.8172 \\ 0 \\ -0.0060 \\ -0.0587 \\ -0.1264 \end{bmatrix}^T$$

Solution of all voltages in modulus (pu and kV) and angle in degrees are given in Table I

TABLE I
SOLUTION IEEE 4-NODE TEST SYSTEM - SINGLE-PHASE BALANCED LOAD [10]

	Mod. (kV)	Mod. (pu)	Angle (deg)
Node 0	12.47	1.0000	+0.0000
Node 1	12.34	0.9893	-0.3486
Node 2	3.91	0.9399	-3.5804
Node 3	3.44	0.8269	-8.7925

Active energy estimated is W_{C3}=582.0pu, and residuals are given by:

$$V_{M0} - V_{C0} = 4.6065E - 05$$
$$P_{M01} - P_{C01} = 0.16002311$$
$$Q_{M01} - Q_{C01} = 0.004146852$$
$$W_{M3} - W_{C3} = 4.6065E - 05$$

In this case, results are very similar to solutions reported in [10] because active energy measured across 720h and a load and a power factor 0.9 and 0.9 respectively, produces a power injection at bus 4 equal to reported in the original power flow solution. As a result, energy measurement has a low residual despite low confidence assigned by the weight matrix. This result is meaningful because it demonstrates that a good energy measurements at users lead to low residuals.

V. CONCLUSION

This paper discuss a new formulation for system state estimation of passive electric distribution networks. A small example is solved for illustration purposes. Results show that proposed formulation is suitable to be applied in order to solve the problem using a general equation formula relating the state of the system, i.e. nodal voltages and injected currents through a general impedance matrix called TRX.

VI. ACKNOWLEDGMENT

This work was supported by MCTII-FONACIT PEI Project 2012000092

APPENDIX A

In the following, the structure $\mathbf{TRX}$ matrix is deduced. First, voltage vector $\mathbf{V}$ is defined according the Kirchhoff Voltage Laws (KVL) as function of branch currents vector $\mathbf{J}$ and line impedances $\mathbf{Z}$:

$$\mathbf{V} = \mathbf{V_0} - \mathbf{T}^T \cdot diag(\mathbf{Z}) \cdot \mathbf{J}$$

where $diag(\mathbf{Z})$ is the diagonal matrix of vector $\mathbf{Z}$.
and, $\mathbf{T}$ is the triangular matrix obtained from Kirchhoff Current Laws (KCL)

$$\mathbf{J} = -\mathbf{T} \cdot \mathbf{I}$$

Voltage equation can be rewritten as::

$$\mathbf{V} = \mathbf{V_0} - \mathbf{T}^T \cdot diag(\mathbf{Z}) \cdot \mathbf{J} = \mathbf{V_0} + \mathbf{T}^T \cdot diag(\mathbf{Z}) \cdot \mathbf{T} \cdot \mathbf{I}$$

Separating real and imaginary parts:

$$\mathbf{V_x} + j\mathbf{V_y} = \mathbf{V_{x0}} + j\mathbf{V_{y0}} - \mathbf{T}^T \cdot (\mathbf{R} + j\mathbf{X}) \cdot (\mathbf{J_x} + j\mathbf{J_y})$$

where,

$$\mathbf{R} = \Re e\{diag(\mathbf{Z})\}; \quad \mathbf{X} = Im\{diag(\mathbf{Z})\}$$
$$\mathbf{J_x} = \mathbf{T} \cdot \mathbf{I_x}; \quad \mathbf{J_y} = \mathbf{T} \cdot \mathbf{I_y}$$

This expression corresponds to Kirchhoff Voltage Laws (KVL) through all nodes and, where $\mathbf{J_x} + j\mathbf{J_y}$ term corresponds to all nodal Kirchhoff Current Laws (KCL). After some algebra, real and imaginary parts can be rewritten as:

$$\mathbf{V_x} = \mathbf{V_{x0}} + \mathbf{T}^T \cdot \mathbf{R} \cdot \mathbf{T} \cdot \mathbf{I_x} - \mathbf{T}^T \cdot \mathbf{X} \cdot \mathbf{T} \cdot \mathbf{I_y}$$
$$\mathbf{V_y} = \mathbf{V_{y0}} + \mathbf{T}^T \cdot \mathbf{X} \cdot \mathbf{T} \cdot \mathbf{I_x} + \mathbf{T}^T \cdot \mathbf{R} \cdot \mathbf{T} \cdot \mathbf{I_y}$$

Voltage drop equations are settled as:

$$\begin{bmatrix} \mathbf{V_x} \\ \mathbf{V_y} \end{bmatrix} - \begin{bmatrix} \mathbf{V_{x0}} \\ \mathbf{V_{y0}} \end{bmatrix} = \begin{bmatrix} \mathbf{T}^T\mathbf{RT} & -\mathbf{T}^T\mathbf{XT} \\ \mathbf{T}^T\mathbf{XT} & \mathbf{T}^T\mathbf{RT} \end{bmatrix} \begin{bmatrix} \mathbf{I_x} \\ \mathbf{I_y} \end{bmatrix}$$

where the $\mathbf{TRX}$ matrix is given by:

$$\mathbf{TRX} = \begin{bmatrix} \mathbf{T}^T\mathbf{RT} & -\mathbf{T}^T\mathbf{XT} \\ \mathbf{T}^T\mathbf{XT} & \mathbf{T}^T\mathbf{RT} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{bmatrix}$$

REFERENCES

[1] F. C. Schweppe and J. Wildes, Power System Static-State Estimation, Part I: Exact model, IEEE Transactions on Power Apparatus and Systems, PAS-89, no. 1, pp. 120-125, 1970.
[2] F. C. Schweppe and D. B. Rom, Power System Static-State Estimation, Part II: Approximate Model, IEEE Transactions on Power Apparatus and Systems, PAS-89, no. 1, pp. 125-130, 1970.
[3] F. C. Schweppe, Power System Static-State Estimation, Part III: Implementation, IEEE Transactions on Power Apparatus and Systems, PAS-89, no. 1, pp. 130-135, 1970.
[4] J. H. Teng and W.-H. E. Liu, Distribution System State Estimation, IEEE Transactions on Power Systems, vol. 10, no. 1, pp. 229-240, 1995.
[5] M. E. Baran and A. W. Kelley, State Estimation for Real-Time Monitoring of Distribution Systems, IEEE Transactions on Power Systems, vol. 9, no. 3, pp. 1601-1609, 1994.
[6] M. E. Baran and A. W. Kelley, A Branch-Current-Based State Estimation Method for Distribution Systems, IEEE Transactions on Power Systems, vol. 10, no. 1, pp. 483-491, 1995.
[7] D. Shirmohammadi, H.W. Hong, A. Semlyen, and G.X. Luo, "A compensation-based power flow method for weakly meshed distribution and transmission networks, IEEE Trans. on Power Deliv., 3, (2), (1988), 753-762
[8] P. Aravindhababu, ,S. Ganapathy, and K.R. Nayar, A novel technique for the analysis of radial distribution systems, Electr. Power Syst. Res., 23, (3) (2001), 167-171
[9] S. Jovanovic and F. Milicevic, Triangular distribution load flow, IEEE Power Eng. Rev., 20, (5), (2000), 60-62
[10] IEEE Distribution System Analysis Subcommittee, IEEE 4 Node Test Feeder, 2006